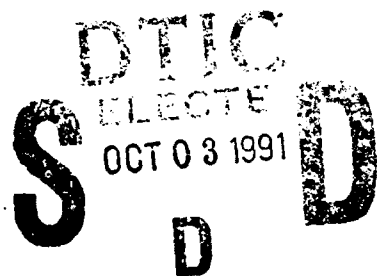


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A Recursive Algorithm for Computing Ray-Parameter Derivatives

by

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A Recursive Algorithm For Computing Ray-parameter Derivatives

Zhenyue Liu

1. INTRODUCTION

Under the assumption of high frequency, wave propagation can be described by the ray method. Many quantities in wave phenomena are characterized by some ray parameters and their derivatives. For example, the slope of arrival time-offset curve is characterized by the emergence angle of a ray; the curvature is characterized by derivative of that angle, and the amplitude of the signal is characterized by the derivative of the taking-off angle. Moreover, when we solve an equation containing ray parameter, we also require analytic expressions for the derivatives. Generally, it is difficult to obtain analytic expressions for these derivatives. We can use finite difference approximation for these derivatives. This type of approximation fails at caustics. And also the accuracy and stability of the calculation are influenced by the smoothness of interfaces and the spacing between adjacent rays.

In this paper, we give a recursive algorithm for ray-parameter derivatives, when the medium is made up of constant-velocity layers separated by arbitrary, smooth interfaces. The algorithm uses derivatives of interfaces, intersection points of rays with the interfaces, and angles of incidence and transmission, all of which are easily computed. A conclusion of these formulas is that no ray caustic exists for transmission or for the reflection from any one of the reflectors when the interfaces are flat.

2. RECURSIVE ALGORITHM

Suppose that an acoustic earth model is made up of constant-velocity layers separated by arbitrary, smooth interfaces. We are going to derive a recursive formula for computing ray-parameter derivatives when a ray propagates through the interfaces.

We consider two interfaces L_1 and L_2 , defined respectively by $z = f_1(x)$ and $z = f_2(x)$. A ray from L_1 arrives at L_2 , with intersection points (x_1, z_1) and (x_2, z_2) . The incident angle from the vertical at L_1 is α_1 and the outgoing angle is α_2 . The angle of the normal at L_1 is θ . When the ray arrives to L_1 , there two possibilities:

reflection and transmission. Suppose that for this ray the initial angle from the vertical is β and the initial depth is z_0 .

Starting from the tangent equation

$$-\frac{df_1}{dx} \big|_{x=x_1} = \tan \theta,$$

the Snell' law

$$\frac{\sin(\alpha_1 - \theta)}{c_1} = \frac{\sin(\alpha_2 - \theta)}{c_2} \text{ for transmission,}$$

$$\alpha_2 - \theta = \theta - \alpha_1 \text{ for reflection,}$$

and the stright line equation

$$x_2 - x_1 = (z_2 - z_1) \tan \alpha_2,$$

and differentiating them with respect to β , we find a recursive algorithm as follows:

$$\frac{d\theta}{d\beta} = -\frac{d^2 f_1}{dx^2} \big|_{x=x_1} \frac{dx_1}{d\beta} \cos^2 \theta. \quad (1)$$

$$\frac{d\alpha_2}{d\beta} = \frac{d\theta}{d\beta} + \frac{c_2 \cos(\alpha_1 - \theta)}{c_1 \cos(\alpha_2 - \theta)} \left(\frac{d\alpha_1}{d\beta} - \frac{d\theta}{d\beta} \right) \text{ for transmission,} \quad (2)$$

or

$$\frac{d\alpha_2}{d\beta} = 2 \frac{d\theta}{d\beta} - \frac{d\alpha_1}{d\beta} \text{ for reflection,} \quad (3)$$

$$(1 - \tan \alpha_2 \frac{df_2}{dx} \big|_{x=x_2}) \frac{dx_2}{d\beta} = \frac{z_2 - z_1}{\cos^2 \alpha_2} \frac{d\alpha_2}{d\beta} + (1 - \tan \alpha_2 \frac{df_1}{dx} \big|_{x=x_1}) \frac{dx_1}{d\beta}. \quad (4)$$

The initial values of the relevant parameters are

$$\frac{d\alpha_1}{d\beta} = 1,$$

$$\frac{dx_1}{d\beta} = \frac{z_1 - z_0}{\cos^2 \alpha_1} / (1 - \tan \alpha_1 \frac{df_1}{dx} \big|_{x=x_1}).$$

3. APPLICATION

Geometry spreading factor $\partial\beta / \partial\xi$

From Docherty (1987), we require the calculation of $\partial\beta / \partial\xi$ in order to obtain the amplitude of the signal. Here,

$$\frac{\partial\beta}{\partial\xi} = 1 / \frac{dx_2}{d\beta} \big|_{x_2=\xi},$$

where β is the taking-off angle of the ray and ξ is the horizontal coordinate of the end of the ray. The factor $dx_2/d\beta$ can be calculated by using equation (4).

Curvature of arrival time-offset curve

Starting from the equation

$$\frac{\partial \tau}{\partial x} = \frac{\sin \alpha}{v},$$

where α is the emergence angle of the ray and v is the velocity, it follows that

$$\frac{\partial^2 \tau}{\partial x^2} = \frac{\cos \alpha}{v} \frac{\partial \alpha}{\partial x}.$$

Since the upper surface is horizontal, $\alpha = \alpha_2$ at $z = 0$. From equations (2) to (4), we can compute $d\alpha_2/d\beta$ and $dx_2/d\beta$ from which we find

$$\frac{\partial \alpha}{\partial x} = \frac{d\alpha_2}{d\beta} \Big|_{\alpha_2=\alpha} / \frac{dx_2}{d\beta} \Big|_{x_2=x},$$

Caustic for flat interfaces

An interesting conclusion of equations (1) to (4) is that when interfaces are flat, no caustic exists. i.e., $\Delta\beta \neq 0$ implies $\Delta x \neq 0$. In fact, for flat interfaces, $d\theta/d\beta = 0$ from equation (1). For the transmission, all coefficients are positive in equations (2) and (4), so $dx_2/d\beta$ is positive. For the reflection, $d\beta_2$ and $z_2 - z_1$ change the sign at same time, so $dx_2/d\beta$ still keeps the positive sign. This implies that rays from two different taking-off angles must not intersect each other for transmission or reflection from one reflector.

4. CONCLUSION

The recursive algorithm has higher accuracy and more stability than the finite difference approximation. Compared the ray tracing, only a little additional computation is required for the recursive algorithm. Furthermore, because of its analytic expression, the recursive algorithm provides a useful tool for the theoretical analysis.

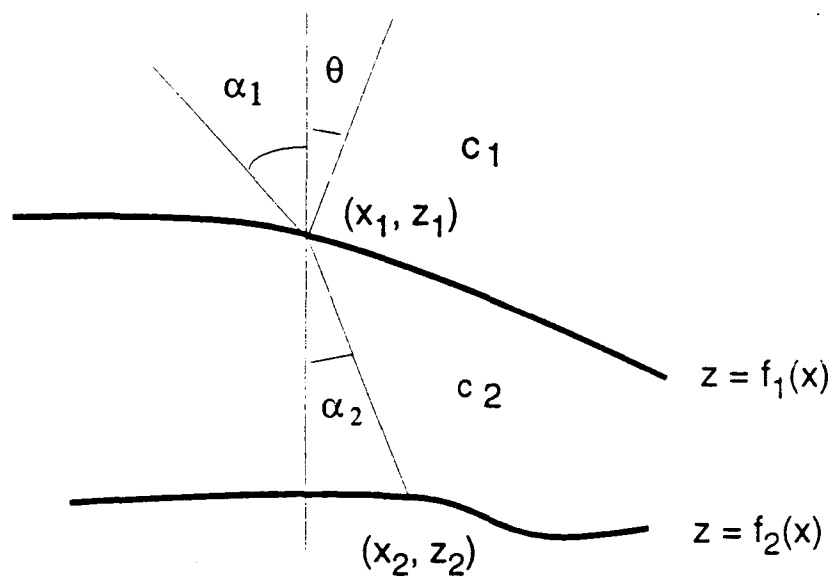
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Docherty, P. C., 1987, Two-and-one-half dimensional common shot modeling: Center for wave phenomena Research Report No. CWP-050.

Transmission



Reflection

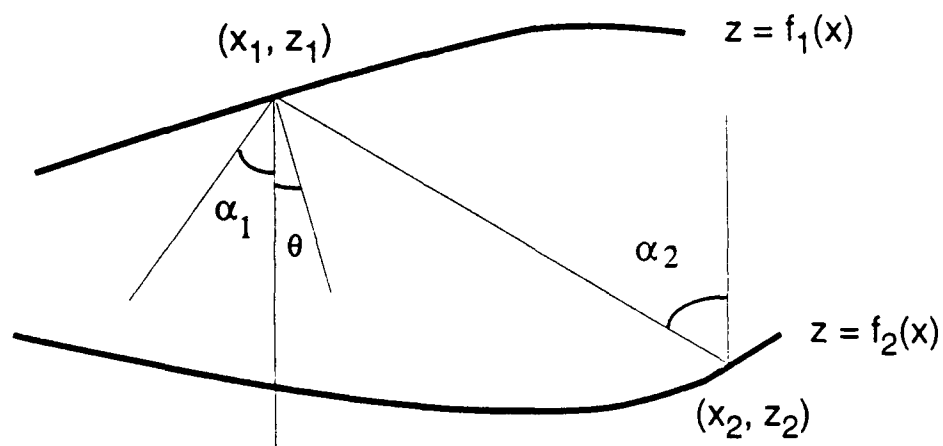


FIG. 1. Paths of arriving ray for transmission and reflection.

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In this paper, a recursive algorithm for ray-parameter derivatives is given when the medium is made up of constant-velocity layers separated by arbitrary, smooth interfaces. The algorithm uses derivatives of interfaces, intersection points of rays with the interfaces, and angles of incidence and transmission, all of which are easily computed. A conclusion of these formulas is that no ray caustic exists for transmission or for the reflection from any one of the reflectors when the interfaces are flat.

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